Course: Applied Mathematics
A.Y.: 2023/2024
Date of classes: 27/11/2023 – 22/12/2023
Final exam: 08/01/2024, from 10.00 to 13.00 (the room will be communicated a.s.a.p.)
Lecturer: Massimiliano Martinelli (martinelli@imati.cnr.it)
Classroom: Eucentre room 1
Last update: 25/10/2023

Week	Date	Lecture hours	Subject	Tot h
1	27/11/23	10:00/13:00	---	
	28/11/23	10:00/13:00	---	
	29/11/23	10:00/13:00	---	
	30/11/23	10:00/13:00	Complex numbers and real analysis	3
	01/12/23	10:00/13:00	Topics on linear algebra. Introduction to Matlab	3
2	04/12/23	10:00/13:00	Eigenvalues and eigenvectors. Matrix diagonalization.	3
	05/12/23	10:00/13:00	Unconstrained optimization.	3
	06/12/23	10:00/13:00	Constrained optimization: Lagrange Multipliers.	3
	07/12/23	10:00/13:00	Constrained optimization: Karush-Kuhn-Tucker.	3
	08/12/23	---	(holiday)	
3	11/12/23	10:00/13:00	Constrained optimization: examples and exercises.	3
	12/12/23	10:00/13:00	Introduction to ODEs	3
	13/12/23	10:00/13:00	First-order linear ODEs (with constant coefficients)	3
	14/12/23	10:00/13:00	Systems of linear ODEs.	3
	15/12/23	10:00/13:00	Stability of linear/nonlinear dynamical systems	3
4	18/12/23	10:00/13:00	L^2 spaces, orthogonal polynomials, Legendre polynomials	3
	19/12/23	10:00/13:00	Least squares approximation	3
	20/12/23	10:00/13:00	Fourier expansion (real and complex form).	3
	21/12/23	10:00/13:00	Fourier transform	3
	22/12/23	10:00/13:00	Fourier transform (cont.); Dirac's delta	3
				48

Note: in case of overlap with other classes, the lecture hours of Friday 01/12/2023 will be moved to 14.00-17.00 of the same day. Further communications will be given by email.

OBJECTIVES: To provide advanced mathematical tools that will be used throughout the rest of the program.

DESCRIPTION: The course is divided into 1+3 chapters as follows.

0. **Prerequisites.** Complex numbers: cartesian, polar and exponential representations; properties and operations. Linear algebra: matrix operations (sum, multiplication, determinant); eigenvalues and eigenvectors. Calculus: differentiation and integration of N-variate real functions.
1. **Optimization of N-variate functions.** Free and constrained optimization of N-variate functions. Lagrange multipliers and KKT conditions. Optimization algorithms (Gradient, Newton, finite differences)
2. **Ordinary Differential Equations (ODE)** Scalar ODEs and system of ODEs. Analytic solutions of linear systems of ODEs (exponential matrix). Study of the harmonic oscillator (damped and with external force). Equilibria of linear and non-linear systems (linearization, Lyapunov's function).
3. **Function approximation and Fourier.** Space of square-summable functions, orthonormal bases and Parseval's identity, Fourier and Legendre expansions, interpolation and least squares approximation. Fourier transform, Dirac's delta.

MATLAB will be used during the classes to provide examples of the discussed topics.

REFERENCES: Class notes made available during the course. For backup and further readings:

- **Optimization of N-variate functions (Ch. 1):** J. Nocedal, S.Wright. Numerical Optimization. Springer;
- **Ordinary Differential Equations (Ch. 2):** G. Teschl, Ordinary Differential Equations and Dynamical Systems, American Mathematical Society; Blanchard, Devaney, Hall. Differential Equations, Cengage Learning.
- **Function approximation, transforms (Ch. 3):** A. Quarteroni, R. Sacco, F. Saleri. Numerical Mathematics. Springer; D. Kammler, A First Course in Fourier Analysis, Cambridge University Press;
- **MATLAB:** MATLAB Primer (https://it.mathworks.com/help/pdf_doc/matlab/learn_matlab.pdf), MATLAB Programming Fundamentals (https://www.mathworks.com/help/pdf_doc/matlab/matlab_prog.pdf),

Italian-speaking students can also use these books:

- **Chapters 1, 2, 3:** Analisi Matematica 2, M. Bramanti, C. Pagani, S. Salsa, Zanichelli ed.;

ASSESSMENT: The final grade will be given after a written exam over the content of the class (theory, exercises).

COURSE WEBSITE: https://elearning.unipv.it/course/view.php?id=6106